General form of β – matrices of the first order wave equations in 16-dimensional representation.

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1. INTRODUCTION

The relativistic theory of free particles of finite mass and noninfinite spin is a well-investigated area [1-4]. Problems however arise concerning the interactions. As usual the system of the first order equations is considered , so as every higher-order equation is reducible to the system of the first order equations. The well-known examples are the Dirac equation and Kemmer-Duffin-Petiau (KDP) equations [1-5]. All these equations contain some NxN matrices, which determine the spin-state structure of the model . Here we are interested in the KDP-like equations, especially the case N=16 . Although in this article we restrict ourselves with the theory of free particles , we compute some general expressions for the further investigations of models with interactions. In spite of the general theory of above-mentioned matrices [1,4] , most of the authors have confined themselves only with some special cases [6-9], and possibly, some important features have been lost. Subsequently we present only the basic elements of the Lorentz-invariant wave equations , in the next three sections we find the general expressions of the above-mentioned matrices with respect to the three different basises (direct product -, Gelfand - and KDP ones) and the last section is dedicated to Hermitianizing matrices in these basises. Here we are not interested in the physical meaning of these results, whereas this is the subject of the next papers.

It is well known [1-4] that the system of equations for a free field of arbitrary spin

$$\left(i\beta^{\mu}\partial_{\mu} - m\right)\psi(x) = 0 \tag{1}$$

(where β_{μ} is a set of four NxN-dimensional matrices, independent of x) is invariant under the homogeneous Lorentz group, if

$$T(\Lambda) \;\; : \quad \psi(x) \to \psi^{'}(x^{'}) = T(\Lambda)\psi(x)$$

$$T^{-1}(\Lambda)\beta_{\mu}T(\Lambda) = \Lambda_{\mu\nu}\beta^{\nu} \;\; \Leftrightarrow \;\; \left[\beta_{\mu,}, S_{\rho\sigma}\right] = i\left(\eta_{\mu\rho}\beta_{\sigma} - \eta_{\mu\sigma}\beta_{\rho}\right), \tag{2}$$

Here the transformation T stands for the finite-dimensional representation of the homogeneous Lorentz group:

$$T \;\; : \;\; SO_{1,3} \ni \Lambda(\omega) \to T(\Lambda) = \exp(-\frac{i}{2}\omega_{\rho\sigma}S^{\rho\sigma})$$

Denoting : $R_i = -\frac{1}{2}\varepsilon_{ijk} S^{jk}, S_i = S^{0i} \quad (i,j,k = 1,2,3)$, one gets :

$$\left.\begin{array}{l}[[\beta_0, S_3], S_3] + \beta_0 = 0 \\ [\beta_0, R_i] = 0\end{array}\right\} \tag{3a}$$

and

$$\beta_i = -i[\beta_0, S_i] \ . \tag{3b}$$

Therefore, it is sufficient to find only β_0, other β_i-s are derivable, using boost-transformations S_i. So for the well known Dirac equation (the representation $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$) the γ_μ matrices satisfy

$$S_{\rho\sigma} = \frac{1}{4}(\gamma_\rho\gamma_\sigma - \gamma_\sigma\gamma_\rho) \ , \quad \gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu} \ , \tag{4}$$

and the field ψ describes a spin-1/2 particle. The 16-dimensional Kemmer-Duffin-Petiau (KDP) representation with properties

$$S_{\rho\sigma} = \beta_\rho\beta_\sigma - \beta_\sigma\beta_\rho \ , \qquad \beta_\rho\beta_\mu\beta_\sigma + \beta_\sigma\beta_\mu\beta_\rho = g_{\mu\rho}\beta_\sigma + g_{\mu\sigma}\beta_\rho \ , \tag{5}$$

consists of three irreducible fields, the first of which is trivial (one-dimensional), while the other two are spin 0 (five-dimensional) and spin 1 (ten-dimensional) fields. There are attempts to describe both, bosons and fermions, using the KDP equations [6-9], but unfortunately, using only the special cases of the β – matrices. Although the KDP-algebra (5) is possible in the case of arbitrary dimension of space-time [5] , here we restrict ourselves with common dimension four .

2. DIRECT PRODUCT BASIS

In the *direct-product* (DP) *basis* $((\frac{1}{2},0)\oplus(0,\frac{1}{2}))\otimes((\frac{1}{2},0)\oplus(0,\frac{1}{2}))$ all the matrices may be expressed via the direct products of the Dirac γ -matrices , so the Lorentz-generators are:

$$S^{\mu\nu} = \frac{1}{2}(\gamma^\mu\gamma^\nu \otimes I + I \otimes \gamma^\mu\gamma^\nu) \tag{6}$$

and the common choise of β - matrices is :

$$\beta^\mu = \frac{1}{2}(\gamma^\mu \otimes I + I \otimes \gamma^\mu) \ . \tag{7}$$

The DP-basis is most favoured in the physical literature , but the spin states are mixed up here. In an another basis (KDP) it comes out, that the choise (7) describes two particles with same mass m , but different spin , 0 and 1.

Direct calculations give that the most general expression of β^0, which satisfies the relativistic invariance conditions (3a) in this basis , is :

$$\beta^0{}_{DP} = \begin{bmatrix}
0 & 0 & z_1 & 0 & 0 & 0 & 0 & 0 & w_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & z_2 & 0 & 0 & z_1-z_2 & 0 & 0 & w_2 & 0 & 0 & w_1-w_2 & 0 & 0 & 0 \\
z_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_5 & 0 & 0 & 0 & 0 & 0 \\
0 & z_4 & 0 & 0 & z_3-z_4 & 0 & 0 & 0 & 0 & 0 & 0 & w_6 & 0 & 0 & w_5-w_6 & 0 \\
0 & 0 & 0 & z_1-z_2 & 0 & 0 & z_2 & 0 & 0 & w_1-w_2 & 0 & 0 & w_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & z_1 & 0 & 0 & 0 & 0 & 0 & w_1 & 0 & 0 \\
0 & z_3-z_4 & 0 & 0 & z_4 & 0 & 0 & 0 & 0 & 0 & 0 & w_5-w_6 & 0 & 0 & w_6 & 0 \\
0 & 0 & 0 & 0 & 0 & z_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_5 \\
w_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & z_5 & 0 & 0 & 0 & 0 & 0 \\
0 & w_4 & 0 & 0 & w_3-w_4 & 0 & 0 & 0 & 0 & 0 & 0 & z_6 & 0 & 0 & z_5-z_6 & 0 \\
0 & 0 & w_7 & 0 & 0 & 0 & 0 & 0 & z_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & w_8 & 0 & 0 & w_7-w_8 & 0 & 0 & z_8 & 0 & 0 & z_7-z_8 & 0 & 0 & 0 \\
0 & w_3-w_4 & 0 & 0 & w_4 & 0 & 0 & 0 & 0 & 0 & 0 & z_5-z_6 & 0 & 0 & z_6 & 0 \\
0 & 0 & 0 & 0 & 0 & w_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & z_5 \\
0 & 0 & 0 & w_7-w_8 & 0 & 0 & w_8 & 0 & 0 & z_7-z_8 & 0 & 0 & z_8 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & w_7 & 0 & 0 & 0 & 0 & 0 & z_7 & 0 & 0
\end{bmatrix} \quad (8)$$

where $z_1,\ldots,z_8, w_1,\ldots,w_8$ are arbitrary 16 parameters. The determinant of this matrix is :

$$\det\beta^0_{DP} = (z_3z_5 - w_3w_5)^3(w_1w_7 - z_1z_7)^3(z_3z_5 - w_3w_5 - 2z_4z_5 + 2w_4w_5 + 4z_4z_6 - 4w_4w_6 - 2z_3z_6 + 2w_3w_6)(w_1w_7 - z_1z_7 - 2w_1w_8 + 2z_1z_8 + 4w_2w_8 - 4z_2z_8 - 2w_2w_7 + 2z_2z_7) \quad (9)$$

In the common case (7) $\det\beta = 0$ and there are no inverse matrices for $\beta - s$. But in the general case it is possible that $\det\beta \neq 0$.

Since every arbitrary 4x4-matrix may be decomposed with respect to the Dirac-basis $\Gamma_s = \{I, \gamma_\mu, \gamma_5, \gamma_5\gamma_\mu, \sigma_{\mu\nu}\}$, it is possible to present general β_μ as in (7) :

$$\beta_\mu = \beta_\mu{}^1 + \beta_\mu{}^2 \qquad \text{, where}$$

$$\beta_\mu{}^1 = a_1(\gamma_\mu \otimes I) + b_1(\gamma_5\gamma_\mu \otimes \gamma_5) + c_1\varepsilon_{\mu\nu\alpha\beta}(\gamma^\nu \otimes \sigma^{\alpha\beta}) + d_1(\gamma^\nu \otimes \sigma_{\mu\nu}) + l_1(\gamma_5\gamma_\mu \otimes I) + f_1(\gamma_\mu \otimes \gamma_5) + k_1\varepsilon_{\mu\nu\alpha\beta}(\gamma_5\gamma^\nu \otimes \sigma^{\alpha\beta}) + e_1(\gamma_5\gamma^\nu \otimes \sigma_{\mu\nu})$$

$$\beta_\mu{}^2 = a_2(I \otimes \gamma_\mu) + b_2(\gamma_5 \otimes \gamma_5\gamma_\mu) + c_2\varepsilon_{\mu\nu\alpha\beta}(\sigma^{\alpha\beta} \otimes \gamma^\nu) + d_2(\sigma_{\mu\nu} \otimes \gamma^\nu) + l_2(I \otimes \gamma_5\gamma_\mu) + f_2(\gamma_5 \otimes \gamma_\mu) + k_2\varepsilon_{\mu\nu\alpha\beta}(\sigma^{\alpha\beta} \otimes \gamma_5\gamma^\nu) + e_2(\sigma_{\mu\nu} \otimes \gamma_5\gamma^\nu) \quad (10)$$

Here $a_1, a_2, \ldots\ldots, e_1, e_2$ are arbitrary 16 constants and

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \qquad \sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right]$$

$\varepsilon_{\mu\nu\alpha\beta}$ - completely antisymmetric unit tensor.

We use spinor representation $\gamma^0 = \sigma^1 \otimes I = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \gamma^k = -i\sigma^2 \otimes \sigma^k = \begin{bmatrix} 0 & -\sigma^k \\ \sigma^k & 0 \end{bmatrix},$

where I is 2x2 unit matrix and σ^k the Pauli matrices.
The relations between the two systems of parameters, (8) and (10), are

$$\left.\begin{aligned}
z_1 &= (a_2 + b_2 - c_2 - id_2) + (l_2 + f_2 - k_2 - ie_2)\\
z_2 &= (a_2 + b_2 + c_2 + id_2) + (l_2 + f_2 + k_2 + ie_2)\\
z_3 &= (a_2 - b_2 + c_2 + id_2) - (l_2 - f_2 + k_2 + ie_2)\\
z_4 &= (a_2 - b_2 - c_2 - id_2) - (l_2 - f_2 - k_2 - ie_2)\\
z_5 &= (a_2 - b_2 - c_2 + id_2) + (l_2 - f_2 - k_2 + ie_2)\\
z_6 &= (a_2 - b_2 + c_2 - id_2) + (l_2 - f_2 + k_2 - ie_2)\\
z_7 &= (a_2 + b_2 + c_2 - id_2) - (l_2 + f_2 + k_2 - ie_2)\\
z_8 &= (a_2 + b_2 - c_2 + id_2) - (l_2 + f_2 - k_2 + ie_2)
\end{aligned}\right\} \qquad (11a)$$

$$\left.\begin{aligned}
w_1 &= (a_1 + b_1 - c_1 - id_1) + (l_1 + f_1 - k_1 - ie_1)\\
w_2 &= (a_1 + b_1 + c_1 + id_1) + (l_1 + f_1 + k_1 + ie_1)\\
w_3 &= (a_1 - b_1 + c_1 + id_1) - (l_1 - f_1 + k_1 + ie_1)\\
w_4 &= (a_1 - b_1 - c_1 - id_1) - (l_1 - f_1 - k_1 - ie_1)\\
w_5 &= (a_1 - b_1 - c_1 + id_1) + (l_1 - f_1 - k_1 + ie_1)\\
w_6 &= (a_1 - b_1 + c_1 - id_1) + (l_1 - f_1 + k_1 - ie_1)\\
w_7 &= (a_1 + b_1 + c_1 - id_1) - (l_1 + f_1 + k_1 - ie_1)\\
w_8 &= (a_1 + b_1 - c_1 + id_1) - (l_1 + f_1 - k_1 + ie_1)
\end{aligned}\right\} \qquad (11b)$$

The members with coefficients a_i, b_i, c_i, d_i correspond to a vector, the members with l_i, f_i, k_i, e_i - to a pseudovector . On the other hand, if β^μ depends from a_i, d_i, f_i, k_i , then the β^0 is hermitian (β^k - antihermitian), if - from l_i, e_i, b_i, c_i , then the β^0 is antihermitian (β^k-hermitian). In the special , symmetrical case $a_1 = a_2 \equiv a, \ldots, e_1 = e_2 \equiv e$ and therefore $w_1 = z_1, \ldots, w_8 = z_8$. By choosing subcase $a = 1/2, b = c = d = l = f = k = e = 0$ one gets the β^μ as in (7). Generally the KDP-algebra (5) is satisfied by $\beta^\mu(a, b, l, f)$ for which

$$\left.\begin{aligned}
a^2 + f^2 - b^2 - l^2 &= \frac{1}{4}\\
af - bl &= 0
\end{aligned}\right\} \qquad (12)$$

The nonsymmetrical parts of β - matrices $\beta(a_k, b_k, l_k, f_k)$, ($k = 1$ or 2) satisfy the Dirac algebra (4), if

$$\left.\begin{aligned}
a_k^2 + f_k^2 - b_k^2 - l_k^2 &= 1\\
a_k f_k - b_k l_k &= 0
\end{aligned}\right\} \qquad (13)$$

For the same expressions of $\beta(a_k, b_k, l_k, f_k)$, ($k = 1$ or 2) β -matrices satisfy the *new algebra* [10]

$$\beta_\rho(\beta_\mu \beta_\nu + \beta_\nu \beta_\mu)\beta_\sigma = 2 g_{\mu\nu} \beta_\rho \beta_\sigma \quad , \qquad (14)$$

but in this case the specific condition

$$\left.\begin{array}{l} a_k^2 + f_k^2 - b_k^2 - l_k^2 = 0 \\ a_k f_k - b_k l_k = 0 \end{array}\right\} \tag{15}$$

takes place. It is clear , that the constraints of the new algebra (14) are weaker than the Dirac's ones .

3. GELFAND BASIS

This basis is used by Gelfand *et al* [4] for building the general theory of the first order relativistically invariant equations . The spin states are separated clearly in this basis . Here we use the *Gelfand basis* (G-basis) in the following ordering :

$$(0,0;0) \oplus (0,0;0) \oplus (\tfrac{1}{2},\tfrac{1}{2};0) \oplus (\tfrac{1}{2},\tfrac{1}{2};0) \oplus (1,0;1) \oplus (0,1;1) \oplus (\tfrac{1}{2},\tfrac{1}{2};1) \oplus (\tfrac{1}{2},\tfrac{1}{2};1) \ ,$$

where $(k,l;j)$ denotes the reduction of the irreducible representation (k,l) of $SO_{1,3}$ to the irreducible representation of SO_3 . In this basis

$$S_j = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -i(K_j V_0) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i(K_j V_0) \\ 0 & 0 & 0 & 0 & -im^j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -im^j & 0 & 0 \\ 0 & 0 & -i(K_j V_0)^+ & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i(K_j V_0)^+ & 0 & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

$$R_j = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m^j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m^j & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m^j & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m^j \end{bmatrix} . \tag{17}$$

Here

$$m^1 = -\frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, m^2 = -\frac{i}{\sqrt{2}} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, m^3 = -\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{18}$$

are the generators of the representation D^1 of the rotation group SO_3 , and

$$K_1 = (1,0,0), K_2 = (0,1,0), K_3 = (0,0,1) \text{ and } V_0 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & -1 \\ i & 0 & i \\ 0 & -\sqrt{2} & 0 \end{bmatrix} \qquad (19)$$

are the Hurley matrices [11].

Now the parameters y_i of β_0 , corresponding to spin 0 , and the parameters x_k, corresponding to spin 1 , are separated and embedded into 4x4-block and 12x12-block respectively :

$$\beta^0{}_G = \begin{bmatrix}
0 & 0 & y_5 & y_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & y_7 & y_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
y_1 & y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
y_3 & y_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_5 & 0 & 0 & x_6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_5 & 0 & 0 & x_6 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_5 & 0 & 0 & x_6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_7 & 0 & 0 & x_8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_7 & 0 & 0 & x_8 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_7 & 0 & 0 & x_8 \\
0 & 0 & 0 & 0 & x_1 & 0 & 0 & x_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & x_1 & 0 & 0 & x_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & x_1 & 0 & 0 & x_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & x_3 & 0 & 0 & x_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & x_3 & 0 & 0 & x_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & x_3 & 0 & 0 & x_4 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \qquad (20)$$

The determinant is :

$$\det \beta_G^0 = (x_1 x_4 - x_2 x_3)^3 (x_6 x_7 - x_5 x_8)^3 (y_1 y_4 - y_2 y_3)(y_6 y_7 - y_5 y_8) \ . \qquad (21)$$

The unitary transformation U, which connects the quantities of DP-basis with ones of G-basis $R_G = U R_{DP} U^+$, has a form

$$U = 1/\sqrt{2} \begin{bmatrix}
0 & p & 0 & 0 & -p & 0 & 0 & 0 & 0 & 0 & 0 & -r & 0 & 0 & r & 0 \\
0 & r & 0 & 0 & -r & 0 & 0 & 0 & 0 & 0 & 0 & p & 0 & 0 & -p & 0 \\
0 & 0 & 0 & s & 0 & 0 & -s & 0 & 0 & -q & 0 & 0 & q & 0 & 0 & 0 \\
0 & 0 & 0 & q & 0 & 0 & -q & 0 & 0 & s & 0 & 0 & -s & 0 & 0 & 0 \\
\sqrt{2}m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & m & 0 & 0 & m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{2}m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2}n & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & n & 0 & 0 & n & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2}n \\
0 & 0 & \sqrt{2}s & 0 & 0 & 0 & 0 & 0 & \sqrt{2}q & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & s & 0 & 0 & s & 0 & 0 & q & 0 & 0 & q & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2}s & 0 & 0 & 0 & 0 & 0 & \sqrt{2}q & 0 & 0 \\
0 & 0 & \sqrt{2}q & 0 & 0 & 0 & 0 & 0 & -\sqrt{2}s & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & q & 0 & 0 & q & 0 & 0 & -s & 0 & 0 & -s & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2}q & 0 & 0 & 0 & 0 & 0 & -\sqrt{2}s & 0 & 0
\end{bmatrix} . \qquad (22)$$

Direct calculations give $\det U = -m^3 n^3 (s^2+q^2)^4 (p^2+r^2)$ and from unitarity $U^+U = I$ we conclude that $m^2 = n^2 = 1, s^2+q^2 = 1, p^2+r^2 = 1$. It is possible to take $m=1, n=-1$ and $s=\cos\theta$, $q=\sin\theta$, $p=\cos\varphi$, $r=\sin\varphi$, so that there are only two independent parameters θ and φ . It is easy to write connections between the parameters of G-basis and DP-basis ones :

$$\left.\begin{aligned}
x_1 &= m(qw_3+sz_3)\\
x_2 &= n(qz_5+sw_5)\\
x_3 &= m(qz_3-sw_3)\\
x_4 &= n(qw_5-sz_5)\\
x_5 &= m(qw_1+sz_1)\\
x_6 &= m(qz_1-sw_1)\\
x_7 &= n(qz_7+sw_7)\\
x_8 &= n(qw_7-sz_7)\\
y_1 &= ps(2z_4-z_3)-pq(2w_4-w_3)-rs(2w_6-w_5)+rq(2z_6-z_5)\\
y_2 &= ps(2w_6-w_5)-pq(2z_6-z_5)+rs(2z_4-z_3)-rq(2w_4-w_3)\\
y_3 &= ps(2w_4-w_3)+pq(2z_4-z_3)-rs(2z_6-z_5)-rq(2w_6-w_5)\\
y_4 &= ps(2z_6-z_5)+pq(2w_6-w_5)+rs(2w_4-w_3)+rq(2z_4-z_3)\\
y_5 &= ps(2z_2-z_1)-pq(2w_2-w_1)-rs(2w_8-w_7)+rq(2z_8-z_7)\\
y_6 &= ps(2w_2-w_1)+pq(2z_2-z_1)-rs(2z_8-z_7)-rq(2w_8-w_7)\\
y_7 &= ps(2w_8-w_7)-pq(2z_8-z_7)+rs(2z_2-z_1)-rq(2w_2-w_1)\\
y_8 &= ps(2z_8-z_7)+pq(2w_8-w_7)+rs(2w_2-w_1)+rq(2z_2-z_1)
\end{aligned}\right\} \qquad (23)$$

The SO_3-invariant $R^2 = R_1^2+R_2^2+R_3^2$ is not diagonal in the DP-basis, but is diagonal in the G-basis, having on the main diagonal 4 values 0 and 12 values $j(j+1)=2$, corresponding to the spin 0 and spin 1 parts respectively.

4. KEMMER-DUFFIN-PETIAU BASIS

In many physical applications [7,8,9] the *KDP-basis* is used , ordered as

$$(0,0)\oplus((\tfrac{1}{2},\tfrac{1}{2})\oplus(0,0))\oplus((1,0)\oplus(0,1)\oplus(\tfrac{1}{2},\tfrac{1}{2})) .$$

Thus the 16-dimensional $\beta-$matrices reduce to the direct sum of 1-,5-,and 10-dimensional $\beta-$matrices in the common case (7) . Notice that in this basis the group-theoretical properties of the equations (1) turn to be explicit and the separation of different physical states takes place. In the KDP-basis the Lorentz generators have the form

$$S_j = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & iK_j & 0 & 0 & 0 & 0 & 0 \\ 0 & iK_j^+ & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -if^j & 0 & 0 \\ 0 & 0 & 0 & 0 & if^j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & iK_j^+ \\ 0 & 0 & 0 & 0 & 0 & 0 & iK_j & 0 \end{bmatrix} \qquad (24)$$

$$R_j = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -if^j & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -if^j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -if^j & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -if^j & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -if^j \end{bmatrix}, \qquad (25)$$

where $\left[f^i, f^j\right] = -\varepsilon_k^{ij} f^k$, $f^k = iV_0 m^k V_0^+$ and V_0, K_j, m^k are given in (18) and (19).

Unitary transformation, which connects the quantities of DP-basis with ones of KDP-basis $R_{KDP} = VR_{DP}V^+$, is :

$$V = 1/2 \begin{bmatrix} 0 & v & 0 & 0 & -v & 0 & 0 & 0 & 0 & 0 & 0 & -v & 0 & 0 & v & 0 \\ 0 & 0 & 0 & it & 0 & 0 & -it & 0 & 0 & it & 0 & 0 & -it & 0 & 0 & 0 \\ 0 & 0 & -it & 0 & 0 & 0 & 0 & it & it & 0 & 0 & 0 & 0 & -it & 0 & 0 \\ 0 & 0 & t & 0 & 0 & 0 & 0 & t & -t & 0 & 0 & 0 & 0 & -t & 0 & 0 \\ 0 & 0 & 0 & it & 0 & 0 & it & 0 & 0 & -it & 0 & 0 & -it & 0 & 0 & 0 \\ 0 & t & 0 & 0 & -t & 0 & 0 & 0 & 0 & 0 & 0 & t & 0 & 0 & -t & 0 \\ u & 0 & 0 & 0 & 0 & -u & 0 & 0 & 0 & 0 & u & 0 & 0 & 0 & 0 & -u \\ iu & 0 & 0 & 0 & 0 & iu & 0 & 0 & 0 & 0 & iu & 0 & 0 & 0 & 0 & iu \\ 0 & -u & 0 & 0 & -u & 0 & 0 & 0 & 0 & 0 & 0 & -u & 0 & 0 & -u & 0 \\ -iu & 0 & 0 & 0 & 0 & iu & 0 & 0 & 0 & 0 & iu & 0 & 0 & 0 & 0 & -iu \\ u & 0 & 0 & 0 & 0 & u & 0 & 0 & 0 & 0 & 0 & 0 & -u & 0 & 0 & -u \\ 0 & iu & 0 & 0 & iu & 0 & 0 & 0 & 0 & 0 & 0 & -iu & 0 & 0 & -iu & 0 \\ 0 & 0 & -iu & 0 & 0 & 0 & 0 & iu & -iu & 0 & 0 & 0 & 0 & iu & 0 & 0 \\ 0 & 0 & u & 0 & 0 & 0 & 0 & u & u & 0 & 0 & 0 & 0 & u & 0 & 0 \\ 0 & 0 & 0 & iu & 0 & 0 & iu & 0 & 0 & iu & 0 & 0 & iu & 0 & 0 & 0 \\ 0 & 0 & 0 & iu & 0 & 0 & -iu & 0 & 0 & -iu & 0 & 0 & iu & 0 & 0 & 0 \end{bmatrix}, \qquad (26)$$

where $\det V = iu^{10}t^5v$ and from $V^+V = I$ it follows that $u^2 = t^2 = v^2 = 1$ (It is possible to take $u = t = v = 1$).

The most general β^0, which satisfies the relativistic invariance conditions (3a) , is in the KDP- basis

$$\beta^0{}_{KDP} = \begin{bmatrix} 0 & \xi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi_2 \\ \xi_3 & 0 & 0 & 0 & 0 & \xi_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \xi_5 & 0 & 0 & \xi_6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi_5 & 0 & 0 & \xi_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi_5 & 0 & 0 & \xi_6 & 0 & 0 & 0 & 0 \\ 0 & \xi_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi_8 \\ 0 & 0 & \xi_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi_{10} & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi_{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & \xi_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi_{10} & 0 \\ 0 & 0 & \xi_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & \xi_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \xi_{13} & 0 & 0 & \xi_{14} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi_{13} & 0 & 0 & \xi_{14} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi_{13} & 0 & 0 & \xi_{14} & 0 & 0 & 0 & 0 \\ \xi_{15} & 0 & 0 & 0 & 0 & \xi_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} . \tag{27}$$

The determinant is

$$\det \beta^0_{KDP} = -(\xi_9\xi_{12} - \xi_{10}\xi_{11})^3(\xi_6\xi_{13} - \xi_5\xi_{14})^3(\xi_3\xi_{16} - \xi_4\xi_{15})(\xi_1\xi_8 - \xi_2\xi_7) \ , \tag{28}$$

where parameters ξ_i be expressed by the use of parameters of DP-basis as follows:

$$\left.\begin{aligned} \xi_1 &= \frac{1}{2} itv(z_1 - w_7 + w_1 - z_7 - 2z_2 + 2w_8 - 2w_2 + 2z_8) \\ \xi_2 &= \frac{1}{2} ivu(z_1 - w_7 - w_1 + z_7 - 2z_2 + 2w_8 + 2w_2 - 2z_8) \\ \xi_3 &= \frac{1}{2} itv(w_3 + z_5 - z_3 - w_3 + 2z_4 + 2w_4 - 2w_6 - 2z_6) \\ \xi_4 &= \frac{i}{2}(2z_4 + 2w_4 + 2w_6 + 2z_6 - z_3 - w_3 - w_5 - z_5) \\ \xi_5 &= \frac{1}{2} iut(w_3 - z_3 - w_5 + z_5) \\ \xi_6 &= \frac{1}{2} ut(z_3 - w_3 - w_5 + z_5) \\ \xi_7 &= \frac{i}{2}(z_1 + w_7 + w_1 + z_7 - 2z_2 - 2w_8 - 2w_2 - 2z_8) \\ \xi_8 &= \frac{1}{2} iut(z_1 + w_7 - w_1 - z_7 - 2z_2 - 2w_8 + 2w_2 + 2z_8) \end{aligned}\right\} \tag{29a}$$

$$\left.\begin{aligned}
\xi_9 &= \frac{1}{2} iut(z_1 + w_7 - w_1 - z_7) \\
\xi_{10} &= \frac{i}{2}(z_1 + w_7 + w_1 + z_7) \\
\xi_{11} &= \frac{1}{2} ut(z_1 - w_7 - w_1 + z_7) \\
\xi_{12} &= \frac{1}{2}(z_1 - w_7 + w_1 - z_7) \\
\xi_{13} &= -\frac{i}{2}(z_3 + w_3 + w_5 + z_5) \\
\xi_{14} &= \frac{1}{2}(z_3 + w_3 - w_5 - z_5) \\
\xi_{15} &= \frac{1}{2} iuv(w_3 - z_3 + w_5 - z_5 + 2z_4 - 2w_4 - 2w_6 + 2z_6) \\
\xi_{16} &= \frac{1}{2} iut(w_3 - z_3 - w_5 + z_5 + 2z_4 - 2w_4 + 2w_6 - 2z_6
\end{aligned}\right\} \qquad (29b)$$

In the concrete fashion (9) , where $a_1 = a_2 \equiv a, \ldots, e_1 = e_2 \equiv e$:

$$\left.\begin{aligned}
\xi_1 &= -2itv(l + f + 3c + 3ie) \\
\xi_3 &= -2itv(l - f + 3c - 3ie) \\
\xi_4 &= 2i(a - b + 3k - 3id) \\
\xi_7 &= -2i(a + b + 3k + 3id) \\
\xi_{10} &= 2i(a + b - k - id) \\
\xi_{12} &= 2(l + f - c - ie) \\
\xi_{13} &= -2i(a - b - k + id) \\
\xi_{14} &= -2(l - f - c + ie) \\
\xi_2 &= \xi_5 = \xi_6 = \xi_8 = \xi_9 = \xi_{11} = \xi_{15} = \xi_{16} = 0
\end{aligned}\right\} \qquad (30)$$

It appears that the reduction $16 = 1 \oplus 5 \oplus 10$ (or, $16 = 5 \oplus 1 \oplus 10$) of the $\beta-$matrices takes place only if β^{μ}-s depend solely on a,b,k,d (or, on l, f, c, e). In the case (30) only the reduction $16 = 6 \oplus 10$ is possible.

A general form of β^{μ}, which satisfies the *weak conditions of discrete symmetries* in N=10 -representation , is given in [12]. The same calculations in the case N=5 give nothing new. Since the case N=1 is trivial, the independent parameters of β in the KDP-basis are :

$$\left.\begin{aligned}
&\xi_1 = \xi_2 = \xi_3 = \xi_5 = \xi_6 = \xi_8 = \xi_9 = \xi_{11} = \xi_{15} = \xi_{16} = 0 \\
&\xi_4 = -\xi_7 = i \\
&\xi_{10} = \frac{i(q+i)x}{2}, \xi_{12} = \frac{(i-q)x}{2}, \xi_{13} = -\frac{q+i}{2qx}, \xi_{14} = \frac{i(i-q)}{2qx}
\end{aligned}\right\} \qquad (31)$$

where q, x - arbitrary complex members. If we demand the validity of (30) and (31) together, then :

$$\left.\begin{aligned}
a &= \frac{1}{2} - 3k \\
b &= -3id \\
l &= -3c \\
f &= -3ie \\
c &= \frac{(q-i)(qx^2-i)}{32qx} \\
d &= \frac{i(q+i)(qx^2+i)}{32qx} \\
k &= \frac{1}{8} - \frac{(q+i)(qx^2-i)}{32qx} \\
e &= \frac{(q-i)(qx^2+i)}{32iqx}
\end{aligned}\right\} \qquad (32)$$

The corresponding parameters in the DP-basis are :

$$\left.\begin{aligned}
z_1 &= w_1 = \frac{ix}{2} \\
z_2 &= w_2 = \frac{1}{4}(1+ix) \\
z_3 &= w_3 = -\frac{i}{2x} \\
z_4 &= w_4 = \frac{1}{4}(1-\frac{i}{x}) \\
z_5 &= w_5 = \frac{1}{2qx} \\
z_6 &= w_6 = \frac{1}{4}(1+\frac{1}{qx}) \\
z_7 &= w_7 = \frac{qx}{2} \\
z_8 &= w_8 = \frac{1}{4}(1+qx)
\end{aligned}\right\} \qquad (33)$$

In the special case, if $q = i, x = -i$ we get the well-known expressions (7) . There are only two free parameters x, q (or, $z_1 \neq 0, z_5 \neq 0$) , all others are expressed via these as follows :

$$\left.\begin{aligned} z_2 &= \frac{1}{4}+\frac{z_1}{2} \\ z_3 &= \frac{1}{4z_1} \\ z_4 &= \frac{1}{4}+\frac{1}{8z_1} \\ z_6 &= \frac{1}{4}+\frac{z_5}{2} \\ z_7 &= \frac{1}{4z_5} \\ z_8 &= \frac{1}{4}+\frac{1}{8z_5} \end{aligned}\right\} \tag{34}$$

One can easily verify that here β^{μ} -s satisfy the KDP algebra condition .

5. HERMITIANIZING MATRIX

The invariant bilinear form in this theory is defined as $\overline{\psi}\psi$, where $\overline{\psi} \equiv \psi^{+}H$. This means [11,12] that there exists a nonsingular Hermitian matrix H such that the transformation $T(\Lambda)$ is H - unitary :

$$T^{+}(\Lambda)H = HT^{-1}(\Lambda) \quad \Leftrightarrow \quad HS_{\mu\nu} - S^{+}_{\mu\nu}H = 0 \tag{35}$$

or

$$HR_k - R_k^{+}H = 0 \quad , \quad HS_k - S_k^{+}H = 0 \quad (k = 1,2,3) \tag{35'}$$

No need to say that because of the non-unitarity of any finite-dimensional representation of $SO_{1,3}$ this form is always indefinite. In the common KDP-theory [1-5] $H = 2\beta_0^2 - I$.

The most general *Hermitianizing matrix* satisfying the equations (35), has the form

$$H_G = \begin{bmatrix} h_1 & h_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ h_{10} & h_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -h_5 & -h_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -h_8 & -h_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & h_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_5 & 0 & 0 & h_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_5 & 0 & 0 & h_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_5 & 0 & 0 & h_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_8 & 0 & 0 & h_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_8 & 0 & 0 & h_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_8 & 0 & 0 & h_6 \end{bmatrix} \tag{36}$$

From the Hermiticity condition it follows that

$$h_4 = h_3^*, h_{10} = h_9^*, h_8 = h_7^*, h_1^* = h_1, h_2^* = h_2, h_5^* = h_5, h_6^* = h_6 \ .$$

The non-degenerancy is satisfied if

$$\det H_G = -h_3^3 h_4^3 (h_5 h_6 - h_7 h_8)^4 (h_1 h_2 - h_9 h_{10}) \neq 0 \ . \tag{37}$$

As to the space inversion operator I_r , [12] it may be presented similarly by choosing

$$h_7 = h_8 = h_9 = h_{10} = 0, h_1 = h_2 = h_5 = h_6 = q_0,$$
$$h_4 = q_0 q, h_3 = q_0 / q \tag{38}$$

and $\det I_r = -q_0^{10}$.

The Hermitianizing matrix H in the DP-basis has the form

$$H_{DP}=\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\tilde{h}_9 & 0 & 0 & 0 & 0 & 0\\
0 & \tilde{h}_1 & 0 & 0 & -\tilde{h}_1 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_9+\tilde{h}_2 & 0 & 0 & \tilde{h}_9-\tilde{h}_2 & 0\\
0 & 0 & \tilde{h}_5 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_5 & 0 & 0 & \tilde{h}_6 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -\tilde{h}_1 & 0 & 0 & \tilde{h}_1 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_9-\tilde{h}_2 & 0 & 0 & \tilde{h}_9+\tilde{h}_2 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\tilde{h}_9\\
0 & 0 & 0 & \tilde{h}_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_6 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_5 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_6 & 0 & 0\\
0 & 0 & \tilde{h}_7 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \tilde{h}_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_8 & 0 & 0 & 0\\
2\tilde{h}_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \tilde{h}_{10}+\tilde{h}_3 & 0 & 0 & \tilde{h}_{10}-\tilde{h}_3 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_4 & 0 & 0 & -\tilde{h}_4 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_7 & 0 & 0 & \tilde{h}_8 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_7 & 0 & 0 & 0 & 0 & 0 & \tilde{h}_8 & 0 & 0\\
0 & \tilde{h}_{10}-\tilde{h}_3 & 0 & 0 & \tilde{h}_{10}+\tilde{h}_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\tilde{h}_4 & 0 & 0 & \tilde{h}_4 & 0\\
0 & 0 & 0 & 0 & 0 & 2\tilde{h}_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{39}$$

where

$$\left.\begin{aligned}
\tilde{h}_1 &= \frac{1}{2}(p^2h_1 + pr(h_{10}+h_9) + r^2h_2)\\
\tilde{h}_2 &= \frac{1}{2}(p^2h_9 + pr(h_2-h_1) - r^2h_{10})\\
\tilde{h}_3 &= \frac{1}{2}(p^2h_{10} + pr(h_2-h_1) - r^2h_9)\\
\tilde{h}_4 &= \frac{1}{2}(p^2h_2 - pr(h_9+h_{10}) + r^2h_1)\\
\tilde{h}_5 &= q^2h_6 + qs(h_7+h_8) + s^2h_5\\
\tilde{h}_6 &= q^2h_8 + qs(h_5-h_6) - s^2h_7\\
\tilde{h}_7 &= q^2h_7 + qs(h_5-h_6) - s^2h_8\\
\tilde{h}_8 &= q^2h_5 - qs(h_7+h_8) + s^2h_6\\
\tilde{h}_9 &= \frac{1}{2}mnh_3\\
\tilde{h}_{10} &= \frac{1}{2}mnh_4
\end{aligned}\right\} \tag{40}$$

and $$\det H_{DP} = -256\tilde{h}_9^3\tilde{h}_{10}^3(\tilde{h}_6\tilde{h}_7-\tilde{h}_5\tilde{h}_8)^4(\tilde{h}_1\tilde{h}_4-\tilde{h}_2\tilde{h}_3)\ . \tag{41}$$

In the usual case (7) $H_{DP}=\gamma_0\otimes\gamma_0$, which is provided by the choise $\tilde{h}_1=\tilde{h}_4=\tilde{h}_5=\tilde{h}_8=0$, $\tilde{h}_6=\tilde{h}_7=1$, $\tilde{h}_2=\tilde{h}_3=\tilde{h}_9=\tilde{h}_{10}=1/2$.

In the KDP-basis the

$$H_{KDP} = \begin{bmatrix}
\hat{h}_1 & 0 & 0 & 0 & 0 & \hat{h}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\hat{h}_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\hat{h}_4 \\
0 & 0 & \hat{h}_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{h}_4 & 0 & 0 & 0 \\
0 & 0 & 0 & \hat{h}_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{h}_4 & 0 & 0 \\
0 & 0 & 0 & 0 & \hat{h}_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{h}_4 & 0 \\
\hat{h}_5 & 0 & 0 & 0 & 0 & \hat{h}_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \hat{h}_7 & 0 & 0 & \hat{h}_8 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{h}_7 & 0 & 0 & \hat{h}_8 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{h}_7 & 0 & 0 & \hat{h}_8 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \hat{h}_8 & 0 & 0 & -\hat{h}_7 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{h}_8 & 0 & 0 & -\hat{h}_7 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{h}_8 & 0 & 0 & -\hat{h}_7 & 0 & 0 & 0 & 0 \\
0 & 0 & \hat{h}_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{h}_{10} & 0 & 0 & 0 \\
0 & 0 & 0 & \hat{h}_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{h}_{10} & 0 & 0 \\
0 & 0 & 0 & 0 & \hat{h}_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{h}_{10} & 0 \\
0 & -\hat{h}_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\hat{h}_{10}
\end{bmatrix} \qquad (42)$$

where

$$\left.\begin{aligned}
\hat{h}_1 &= \frac{1}{2}\left[p^2\left(h_1 + h_2 - h_9 - h_{10}\right) + r^2\left(h_1 + h_2 + h_9 + h_{10}\right) + 2pr\left(h_1 - h_2\right)\right] \\
\hat{h}_2 &= \frac{1}{2}vt\left[p^2\left(h_1 - h_2 + h_9 - h_{10}\right) - r^2\left(h_1 - h_2 - h_9 + h_{10}\right) + 2pr\left(h_9 + h_{10}\right)\right] \\
\hat{h}_3 &= \frac{1}{2}\left[q^2\left(h_5 + h_6 - h_7 - h_8\right) + s^2\left(h_5 + h_6 + h_7 + h_8\right) - 2qs\left(h_5 - h_6\right)\right] \\
\hat{h}_4 &= \frac{1}{2}ut\left[-q^2\left(h_5 - h_6 + h_7 - h_8\right) + s^2\left(h_5 - h_6 - h_7 + h_8\right) + 2qs\left(h_7 + h_8\right)\right] \\
\hat{h}_5 &= \frac{1}{2}vt\left[p^2\left(h_1 - h_2 - h_9 + h_{10}\right) - r^2\left(h_1 - h_2 + h_9 - h_{10}\right) + 2pr\left(h_9 + h_{10}\right)\right] \\
\hat{h}_6 &= \frac{1}{2}\left[p^2\left(h_1 + h_2 + h_9 + h_{10}\right) + r^2\left(h_1 + h_2 - h_9 - h_{10}\right) - 2pr\left(h_1 - h_2\right)\right] \\
\hat{h}_7 &= \frac{1}{2}mn\left(h_4 + h_3\right) \\
\hat{h}_8 &= \frac{1}{2}mn\left(h_4 - h_3\right) \\
\hat{h}_9 &= \frac{1}{2}ut\left[-q^2\left(h_5 - h_6 - h_7 + h_8\right) + s^2\left(h_5 - h_6 + h_7 - h_8\right) + 2qs\left(h_7 + h_8\right)\right] \\
\hat{h}_{10} &= \frac{1}{2}\left[q^2\left(h_5 + h_6 + h_7 + h_8\right) + s^2\left(h_5 + h_6 - h_7 - h_8\right) + 2qs\left(h_5 - h_6\right)\right]
\end{aligned}\right\} \qquad (43)$$

and det $_{KDP} = -({}^{2}_{7} + {}^{2}_{8} \; {}^{3} \; {}_{4\;9} - {}_{3\;10} \; {}^{4} \; {}_{1\;6} - {}_{2\;5}$ (44)

Finally let us note that these H - matrices play the important role in the investigation of the discrete symmetries of equations (1) . The *weak conditions of discrete symmetries* are presented in the previous work [12].

7. CONCLUSIONS AND ACKNOWLEDGEMENTS

We presented some general expressions for the β - matrices of the first order wave equations in the 16-dimensional representation , using three different basises. We hope that these representations allow to construct a more realistic theory of elementary particles and their interactions. Of course, this model must contain some nonlinearity . Except the cases (12) - (14) , we have not concretized the algebra of β - matrices . The general 16-component theory of the massive single spin-1 particle , using the new algebra (14), will be published in the nearest future. We hope, that the 16-dimensional representation will enable to embrace successfully both the bosons and fermions within the framework of some realistic model.

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